Workshop on Stochastic Dynamic Games and Related Topics

March 22–24, 2023, Kiel

Sponsors

Contents

About

SDG

This workshop will cover recent advances in the mathematical theory of dynamic stochastic games and related stochastic control problems. The possible areas of application are broad. The goal is to foster existing collaborations and to identify and explore directions for future research.

Local Organizers & Contact Information

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Timetable

CT: Contributed Talk, IS: Invited Speaker

Wednesday, March 22 – Room R.EG.024

8:45–09:00	**Registration (in the foyer)**		
09:00–09:05	**Opening**		
09:05–09:55	IS	**Berenice Neumann** Trier, Germany	Stationary Equilibria of Mean Field Games with Finite State and Action Space
09:55–10:30	CT	**Christian Fabian** Darmstadt, Germany	Mean Field Games on Weighted and Directed Graphs via Colored Digraphons
10:30–11:00	**Coffee**		
11:00–11:50	IS	**Stefan Ankirchner** Jena, Germany	The role of correlation in diffusion control games
11:50–12:25	CT	**Julian Wendt** Jena, Germany	Large ranking games with diffusion control
12:25–14:00	**Lunch**		
14:00–14:50	IS	**Kristoffer Lindensjö** Stockholm, Sweden	Stochastic control and stopping games with unknown competition
14:50–15:25	CT	**Boy Schultz** Kiel, Germany	On a time-inconsistent optimal stopping problem with expectation constraint
15:25–16:00	**Coffee**		
16:00–16:35	CT	**Topias Tolonen** Uppsala, Sweden	Hiring and firing – a stochastic game with incomplete information
16:35–17:25	IS	**Luis Alvarez Esteban** Turku, Finland	The Impact of Ambiguity on the Optimal Exercise Timing of Integral Option Contracts

Link to abstracts:

Thursday, March 23 – Room R.EG.024

09:00–09:50	IS	**Nicole Bäuerle** Karlsruhe, Germany	Time-consistency in the mean-variance problem: A new perspective
09:50–10:25	CT	**Kerem Ugurlu** Astana, Kazakhstan	Dynamic Programming Equations in AVaR
10:25–11:00	**Coffee**		
11:00–11:50	IS	**Jan Palczweski** Leeds, UK	Cancellable American options under negative discounting
11:50–12:25	CT	**Yuqiong Wang** Uppsala, Sweden	Dynkin ghost games with consolation
12:25–14:00	**Lunch**		
14:00–14:50	IS	**Johannes Muhle-Karbe** London, UK	Nash Equilibria with Nonlinear Price Impact
14:50–15:25	CT	**Tamara Göll** Karlsruhe, Germay	Nash equilibria for relative investors under linear price impact
15:25–16:00	**Coffee**		
16:00–16:35	CT	**Wiete Valettt** Kiel, Germany	The general (p:q)-Game in Hypergraphs
16:35–17:10	CT	**Christopher Lorenz** Frankfurt, Germany	Insider trading in discrete time Kyle games
19:00	**Conference Dinner**		

- The conference dinner takes place at

  Lagom
  Düsternbrookerweg 38

- We will meet at 18:20 in front of the math department to walk to the restaurant. You are welcome to join us.

- You are invited for the meal but drinks have to be paid by yourself.

Link to abstracts:

Friday, March 24 – Room R.EG.003/R.EG.004

09:00–09:50	IS	**Christoph Belak** Berlin, Germany	Principal-Agent Games of Optimal Stopping with Applications to the Energy Transition
09:50–10:25	CT	**E. Emanuel Rapsch** Berlin, Germany	Stochastic extensive form games
10:25–11:00	**Coffee**		
11:00–11:35	CT	**Niklas Dexheimer** Aarhus, Denmark	On Lasso and Slope drift estimators for Lévy-driven Ornstein–Uhlenbeck processes
11:35–12:10	CT	**Henrik Valett** Kiel, Germany	Parameter estimation for polynomial processes
12:10–12:15	**Closing**		

Link to abstracts:

List of Abstracts – Talks

Wednesday, March 22

Stationary Equilibria of Mean Field Games with Finite State and Action Space

Berenice Neumann IS

University of Trier, Germany

Mean field games formalize dynamic games with a continuum of players and explicit interaction, where the players can have heterogeneous states. Many economic examples have been formulated as mean field games with finite state and action space. In this talk we will introduce stationary equilibria for these games. We prove existence of these equilibria under mild assumptions and provide a semi-explicit characterization of all equilibria. Thereafter, we show that stationary equilibria are under certain conditions limit points of a learning process, namely, the myopic adjustment process. Moreover, we discuss stability of the equilibria with respect to model perturbations. Finally, we present some results on the relation of dynamic and stationary equilibria for an example.

Mean Field Games on Weighted and Directed Graphs via Colored Digraphons

Christian Fabian

Technische Universität Darmstadt, Germany

Multi-agent systems are in general hard to model and control due to their complex nature involving many individuals. Numerous approaches focus on empirical and algorithmic aspects of approximating outcomes and behavior in multi-agent systems and lack a rigorous theoretical foundation. Graphon mean field games (GMFGs) on the other hand provide a mathematically well-founded and numerically scalable framework for a large number of connected agents. In standard GMFGs, the connections between agents are undirected, unweighted and invariant over time. Our paper introduces colored digraphon mean field games (CDMFGs) which allow for weighted and directed links between agents that are also adaptive over time. Thus, CDMFGs are able to model more complex connections than standard GMFGs. Besides a rigorous theoretical analysis including both existence and convergence guarantees, we employ the online mirror descent algorithm to learn equilibria. To conclude, we illustrate our findings with an epidemics model and a model of the systemic risk in financial markets.
The paper will be published in the IEEE Control Systems Letters:
`https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=9973407`

The role of correlation in diffusion control games

Stefan Ankirchner IS

University of Jena, Germany

In this talk, we describe a stochastic differential game with two players, where each player can control the diffusion intensity of an individual dynamic state process. We suppose that the Brownian motions, driving the players' state equations, are correlated. At a deterministic finite time horizon the players receive a reward depending on their state processes' difference. As long as the correlation of the Brownian motions does not exceed a certain bound there exists a saddle point and the game has a value. For correlations exceeding this bound, however, the game does not have a value. To overcome this issue we introduce the notion of relaxed controls that may be regarded as mixed controls in the differential game setting. We prove that in this class of controls there exists a saddle point and the game has a value for all correlations.
The talk is based on joint work with Nabil Kazi-Tani and Julian Wendt.

Large ranking games with diffusion control

Julian Wendt

University of Jena, Germany

In this talk we consider a symmetric stochastic differential game where each player can control the diffusion intensity of an individual dynamic state process, and the players whose states at a deterministic finite time horizon are among the best $\alpha \in (0,1)$ of all states receive a fixed prize. Within the mean field limit version of the game we compute an explicit equilibrium, a threshold strategy that consists in choosing the maximal fluctuation intensity when the state is below a given threshold, and the minimal intensity else. We show that for large n the symmetric n-tuple of the threshold strategy provides an approximate Nash equilibrium of the n-player game. We also derive the rate at which the approximate equilibrium reward and the best response reward converge to each other, as the number of players n tends to infinity. Finally, we compare the approximate equilibrium for large games with the equilibrium of the two player case.

Stochastic control and stopping games with unknown competition

Kristoffer Lindensjö IS

Stockholm University, Sweden

In most games it is assumed that each player is aware of the existence of the other competing player(s). In contrast to the standard setup, we consider games with the feature of *unknown competition*.
In particular, we consider stochastic control and stopping games where one player does not know if the other player is active or not. Any interesting (Nash) equilibrium in such a setting should then naturally involve at least one player continually updating the conditional probability of the existence of the other player. Accordingly, we develop a stochastic filtering method to find equilibria for games of this type.
The talk is based on joint work with Andi Bodnariu (working paper), and Erik Ekström and Marcus Olofsson [1].

[1] E. Ekström, K. Lindensjö, and M. Olofsson. How to detect a salami slicer: A stochastic controller-and-stopper game with unknown competition. *SIAM J. Control Optim.*, 60(1):545–574, 2022.

On a time-inconsistent optimal stopping problem with expectation constraint

Boy Schultz

Kiel University, Germany

We consider the problem of optimally stopping an Itô diffusion with a stopping time satisfying an expectation constraint, which turns out to be a time-inconsistent optimal stopping problem. We present a game-theoretic approach and introduce mixed strategy stopping times as well as equilibrium strategies. We derive a verification theorem together with properties of candidate stopping times for systematic guessing. Finally we provide some illustrating examples.

Hiring and firing – a stochastic game with incomplete information

Topias Tolonen

Uppsala University, Sweden

We study a strategic game between an employer and a potential employee, where the employee has private information regarding their production capacity. At the initial stage, the employee communicates a salary claim, after which their true production capacity is gradually revealed to the employer as the unknown drift of a Brownian motion representing the revenues generated by the employee. Subsequently, the employer has the possibility to reject the salary claim or choose a time to fire the employee in case the estimated production capacity understeps the salary. We set-up a Perfect Bayesian Equilibrium with mixed strategies and discuss the consequences of the employee wanting or not wanting to reveal their true production capacity. While the game is rather simplistic with only two possible types for the employee and two possible salary claims, which are relative to the types of the employee, it serves as a benchmark for more involved problems. Extensions of the set-up are also discussed.
This submission is based on a joint work with Erik Ekström.

The Impact of Ambiguity on the Optimal Exercise Timing of Integral Option Contracts

Luis Alvarez Esteban IS

University of Turku, Finland

We consider the impact of ambiguity on the optimal timing of a class of two-dimensional integral option contracts when the exercise payoff is a positively homogeneous measurable function. Hence, the considered class of exercise payoffs includes discontinuous functions as well. We identify a parameterized family of excessive functions generating an appropriate class of supermartingales for the considered problems and then express the value of the optimal policy as well as the worst case measure in terms of these processes. The advantage of our approach is that it reduces the analysis of the multidimensional problem to the analysis of an ordinary one-dimensional static optimization problem. In that way it simplifies earlier treatments of the problem without ambiguity considerably. We also illustrate our findings in explicitly parameterized examples.
The study is an ongoing joint research effort with Sören Christensen.

Thursday, March 23

Time-consistency in the mean-variance problem: A new perspective

Nicole Bäuerle IS

Karlsruhe Institute of Technology, Germany

We investigate discrete-time mean-variance portfolio selection problems viewed as a Markov decision process. We transform the problems into a new model with deterministic transition function for which the Bellman optimality equation holds. In this way, we can solve the problem recursively and obtain a time-consistent solution, that is an optimal solution that meets the Bellman optimality principle. We apply our technique for solving explicitly a more general framework. The talk is based on joint work with Anna Jaśkiewicz.

Dynamic Programming Equations in AVaR

Kerem Ugurlu

Nazarbayev University, Kazakhstan

It is well known that the coherent risk measures are time-inconsistent. In particular, the Bellman dynamic programming equations can not be derived when using these kind of operators.
It is shown in [1] that by aggregating the state and keeping specific information from the history, called 'sufficient statistic', one can still give time-consistent strategies. In this talk, we elaborate this idea further and write the corresponding dynamic programming equations of the aggregated states and actually solve/compute the corresponding problems. Numerical examples are also illustrated to elaborate the corresponding algorithms.

[1] N. Bäuerle and J. Ott. Markov decision processes with average-value-at-risk criteria. *Math. Methods Oper. Res.*, 74(3),361–397, 2011.

Cancellable American options under negative discounting

Jan Palczweski IS

University of Leeds, UK

Cancellable American options, also known as game options or Israeli options, are American-style derivatives which give the writer the right to terminate the contract for a fixed penalty. I will talk about perpetual cancellable American put options on an asset whose dynamics follow exponential spectrally negative Levy process. The price and optimal strategies of the buyer and the writer can be deduced from the solution of a corresponding Dynkin game. The new feature of the model is the negative interest rate which brings in difficulties (the payoff grows exponentially fast in time) and interesting strategies. We employ fully probabilistic arguments to argue the existence of the value and of the optimal strategies and characterise explicitly their form. We also prove smooth fit at boundaries of stopping sets enabling their numerical identification.

Dynkin ghost games with consolation

Yuqiong Wang

Uppsala University, Sweden

Some companies want to introduce their new products to the market. Delaying the introduction might increase the quality and profit, but possible competitors might do the same earlier and gain most of the market share. In this article, we study such stopping games with a consolation prize: the second stopper gets rewarded less, and the competitor might not exist. We give a verification result for a Nash equilibrium with randomized strategies. We also state sufficient conditions under which we can solve for the equilibrium value of both players by solving a differential equation system.
This is joint work with Tiziano De Angelis and Erik Ekström.

Nash Equilibria with Nonlinear Price Impact

Johannes Muhle-Karbe IS

Imperial College London, UK

One ubiquitous way in which traders interact in financial markets is through the price impact their order executions have on the quoted prices. A rich literature has emerged that studies the corresponding Nash equilibria under the simplifying assumption that impact is linear. Yet, virtually all empirical studies agree that impact is in fact only approximately linear for very small orders, but follows a square-root law for larger trade sizes. In this talk, we discuss what can be said about traders‘ individual optimization problems in this context, and how such results can be concatenation into a Nash equilibrium in a second step.
Joint work in progress with Kevin Webster.

Nash equilibria for relative investors under linear price impact

Tamara Göll

Karlsruhe Institute of Technology, Germany

We analyze the optimal investment behavior of n agents trading in a Black-Scholes type financial market where the average investment of the n agents affects the drift of the stock price processes linearly. The objective function of a single agent is given in terms of its own as well as the terminal wealth of the other $n-1$ agents. In this context, we determine the unique constant Nash equilibrium by solving an auxiliary problem containing only one investor.
Finally, we compare the Nash equilibrium to the special cases with no price impact or no relative criterion within the utility function. If time permits, we will also briefly consider the case of a nonlinear price impact.
This talk is based on joint work with Nicole Bäuerle.

The general (p:q)-Game in Hypergraphs

Wiete Valett

Kiel University, Germany

We introduce the (p:q)-Game played on a hypergraph $H = (V, E)$ with $|E| = m$ by two players, Balancer and Unbalancer. In each round of the game Balancer selects p elements of the vertex set V before Unbalancer selects q elements of the vertex set V. The game is played until all vertices are selected or one player has achieved his win condition. Balancer's aim is to have around $\frac{p}{p+q}|e_j|$ vertices in every edge $e_j \in E$. Unbalancer tries to prevent Balancer from achieving this goal. Balancer's winning condition can be formulated in the following way. For every edge $e_j \in E$ let $b_j \geq 0$ be the allowed deviation. Balancer wins, if his selected number of vertices t_j in edge e_j satiesfies $\left|t_j - \frac{p}{p+q}|e_j|\right| \geq b_j$ for all edges e_j. We are looking for the smallest possible deviation b_j. Alon et al. (2005) proved that for the (1:1)-Game $b_j \geq \sqrt{2\ln(2m)|e_j|}$. They posed the analysis of the general (p:q)-Game as a challenging open problem. With a special focus on the case $p = q$ we will analyse Balancer's strategy and winning condition for large enough deviation.
Based on joint work with Anand Srivastav.

Insider trading in discrete time Kyle games

Christopher Lorenz

Goethe University Frankfurt, Germany

We present a discrete time version of Kyle's classic model of insider trading. The model has three kinds of traders: an insider, random noise traders, and a market maker. The insider aims to exploit her informational advantage while using the noise trades as camouflage to remain unobserved by the market maker.
In a first step, we show how the multi-period model with finitely many pure strategies can be reduced to a (static) social system in the sense of Debreu (1952) and prove the existence of an equilibrium. Then, we prove the existence of an equilibrium for the game with a continuum of actions, by considering an approximating sequence of games with finitely many actions.
Joint work with Christoph Kühn.

Friday, March 24

Principal-Agent Games of Optimal Stopping with Applications to the Energy Transition

Christoph Belak IS

TU Berlin, Germany

We consider a finite number of agents facing an irreversible investment problem with competition. The agents are assumed to have the option to switch from an old, existing technology for production to a new, more sustainable technology. Returns on investments depend on the demand for the new technology, wich gradually increases in time, and the number of agents having switched to the new technology. In addition, we assume the presence of a principal who can incentivise the adoption of the new technology through subsidies and taxation, leading to a principal-agent problem of optimal stopping with information asymmetry. We apply our model to address the question of how a government can accelerate the transition from internal combustion engines to electric motors in the automotive industry.

Stochastic extensive form games

E. Emanuel Rapsch

TU Berlin, Germany

The extensive form is a fundamental object of dynamic game theory. Essentially, it characterises a game through a game tree, a set of players and a structure of choices available to each player (going back to von Neumann–Morgenstern ‘44, Kuhn ‘50, ‘53, for recent developments see e.g. Alós-Ferrer–Ritzberger ‘05, ‘08). Going back to Shapley ‘53, stochastic games are often modelled using this formalism by adding a (virtual) “nature” player. That way, any relevant “discrete time” noise can be included. But when it comes to, say Brownian motion, even the advanced continuous time extensive form frameworks by Stinchcombe ‘92 and Alós-Ferrer–Kern ‘15 do not apply. This is somewhat surprising since the stochastic differential games literature has dealt with Brownian motion (and more…) for a long time. This works because that literature essentially ignores the issue of extensive forms, and it does so for the (real) “personal” players as well – at the expense of a lack of decision-theoretic interpretability. In this talk, I would like to explain a way of reconciling both approaches. The idea is on the one hand to agree that “nature” does not decide, and rather acts as a one-shot lottery together with a dynamically updating oracle; but on the other hand to stipulate that “personal” players take decisions in an extensive form adapted to the oracles. The “adaptedness” will arise as a consequence of the extensive form structure, rather than as a primitive (as e.g. in Riedel–Steg ‘17).
Part of ongoing doctoral research supervised by Christoph Belak (TU Berlin).

On Lasso and Slope drift estimators for Lévy-driven Ornstein–Uhlenbeck processes

Niklas Dexheimer

Aarhus University, Denmark

We investigate the problem of estimating the drift parameter of a high-dimensional Lévy-driven Ornstein–Uhlenbeck process under sparsity constraints. It is shown that both Lasso and Slope estimators achieve the minimax optimal rate of convergence (up to numerical constants), for tuning parameters chosen independently of the confidence level, which improves the previously obtained results for standard Ornstein–Uhlenbeck processes. The results are nonasymptotic and hold both in probability and conditional expectation with respect to an event resembling the restricted eigenvalue condition.

Parameter estimation for polynomial processes

Henrik Valett

Kiel University, Germany

We consider parameter estimation for discretely observed generic polynomial (and in particular affine) Markov processes, relying on a quasi-likelihood approach. Specifically, we consider polynomial martingale estimating functions up to a given degree. Within this class the Heyde-optimal estimating function can be computed in closed form. This allows to derive consistency and asymptotic normality, based on results of [1] and the ergodic theory for Markov processes.
Joint work with J. Kallsen (Kiel University).

[1] M. Sørensen. Estimating functions for diffusion-type processes. In *Statistical methods for stochastic differential equations*, Volume 124 of *Monogr. Statist. Appl. Probab.*, 1-107, 2012.

List of Participants

Alvarez Esteban, Luis	Turku, Finland
Ankirchner, Stefan	Jena, Germany
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Vetter, Mathias	Kiel, Germany
Wang, Yuqiong	Uppsala, Sweden
Wendt, Julian	Jena, Germany
Yu, Fan	Kiel, Germany

Useful Information

The **workshop** takes place at the

Department of Mathematics
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Heinrich-Hecht-Platz 6
24118 Kiel.

For the **registration** please come to the foyer.

Talks will be held in

- room R.EG.024 (ground floor) on Wednesday and Thursday,
- room R.EG.003/R.EG.004 (ground floor) on Friday.

Coffee breaks are held in the foyer and in room R.EG.025 (ground floor).

For **discussions** you can use the common room 02.036 or the office 02.032 (both second floor). For the latter you need a key which can be obtained from Sören Christensen.

Please **upload your slides** in time before the start of your session to the following cloud folder. Please make sure that the file name is of the following form: day_time_lastname.pdf, e.g., Wednesday_14_50_Schultz.pdf.

Link: `https://cloud.rz.uni-kiel.de/index.php/s/icY6tJjcQb2tfkZ`

Eduroam will be available during the conference.

The **menu** of the **canteen "Mensa I"** can be found here:
Link: `https://studentenwerk.sh/en/mensaplandruck?ort=1&mensa=1`